Numerical Analysis of Surface-Wave Scattering by Finite Periodic Notches in a Ground Plane

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Abstract —**Surface-wave scattering by finite periodic notches loaded in a ground plane is investigated in terms of a full-wave theory. The analytical method presented is based on a spectral-domain analysis where the sampling theorem is applied in order to discretize the final equation to be solved. Numerical calculations are carried out for reflected, transmitted, and radiated waves. The numerical results show that maximum reflection and radiation occur at frequencies somewhat different from those expected from the Bragg condition based on a first-order perturbation theory.**

I. Introduction

PERIODIC LOADING in various kinds of waveguides exhibits an interesting phenomenon called Bragg diffraction. Because of the nonuniform nature of the guides, mode conversion into various other modes occurs. When the periodicity is chosen such that the Bragg condition is satisfied for two specific modes, the mode conversion between them is prominent while that between any others is negligibly small. Therefore, if one of them is an incident surface wave and the other is a reflected wave, the mechanism suggests the possibility of devising an effective mode filter by such periodic loading of the dielectric guide. For an effective leaky wave antenna, the situation is much the same if the second mode is the radiation field with regard to an open-type dielectric guide.

Most work concerning this problem has employed approximate methods such as perturbation or coupled-mode theory [1]. However, only a few investigations in terms of the full-wave theory have appeared so far on finite periodic structures [2]–[5]. In this paper, we deal with finite periodic notches in a ground plane covered by a dielectric slab, which is a natural extension of the single-notch case [6]. Similar structures have already been treated by other researchers in connection with a mode launcher or a dielectric image line-array antenna [7] as well as a grating coupler [8]. However, their analyses have been based on approximate methods, such as the equivalent-circuit representation calculated by a plane resonator model [7] and the second-order perturbation theory [8]. The end effect taking place near $z=0$ and $z=(N-1)D$ in Fig. 1 has also been neglected. The major motivation of the present analysis stems from an interest in making clear the surface-wave scattering by the discontinuities due to the finite periodic notches. This structure is simple but of great importance in practical application to mode filters or leaky wave antennas. This is because the discontinuities can be enhanced by adjusting the depth of the notches appropriately; hence, an effective conversion to reflected or radiated waves may be expected for a small number of notches.

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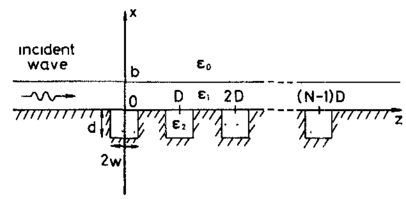

Fig. 1. Geometry of the problem.

In this paper, we propose a new numerical method based on the spectral-domain analysis, combined with the sampling theorem, in order to treat finite periodic structures. The basic idea of the present method is to band-limit a widespread spectral function in the spectral domain by a convolution integral, with the sampling function as a weighting function in relation to a finite length in the space domain [9]. Thus, the sampling theorem ensures that the final equation to be solved can be discretized in the spectral domain for numerical calculations. This new method has already been applied successfully to the infinite periodic structure of a plane grating [10], [11]. However, in this case, Floquet's theorem can be applied, and hence it is sufficient to consider only one period. In the case of a finite structure, on the other hand, the numerical analysis is more cumbersome since we have to take each period into account separately. It is shown that the new method can also be applied to a fairly large number of periodic notches by use of an iterative computation.

II. Formulation

Fig. 1 shows the geometry of the problem, where the structure is uniform in the y direction. Finite periodic notches of width $2w$ and depth d are spaced a distance D from each other with center at

$$z=nD \qquad (n=0,1,2,\cdots,N-1) \tag{1}$$

where N is the number of notches. We consider here only

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the TE excitation in order to minimize the details. In this case, Maxwell's equations can be written as follows:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+\kappa_i^2\right)E_y(x,z)=0 \tag{2}$$

$$\kappa_i=\omega\sqrt{\epsilon_i\mu_0} \qquad (i=0,1,2) \tag{3}$$

where $i=0$, 1, and 2 corresponds, respectively, to free space, the dielectric layer above the ground plane, and the notches. Also,

$$(H_x,H_z)=\left(\frac{1}{j\omega\mu_0}\cdot\frac{\partial E_y}{\partial z},\frac{j}{\omega\mu_0}\cdot\frac{\partial E_y}{\partial x}\right). \tag{4}$$

In these equations, the time dependence $e^{j\omega t}$ is assumed, and all the other field components are zero.

The incident wave is a dominant TE surface wave mode given by

$$\begin{aligned}E_y^i(x,z)&=N(\zeta)e^{-jk_0(x-b)-j\zeta z}\big|_{\zeta=\zeta_1} \qquad (x>b)\\ &=N(\zeta)\frac{\sin kx}{\sin kb}e^{-j\zeta z}\bigg|_{\zeta=\zeta_1} \qquad (0<x<b)\end{aligned} \tag{5}$$

where ζ_1 is the propagation constant of the dominant mode. The propagation constants, including higher modes, are determined by

$$G(\zeta_s)=0 \qquad (s=1,2,\cdots,M) \tag{6}$$

where M is the number of surface wave modes supported on the left uniform waveguide. The kernel function is defined by

$$G(\zeta)=jk_0+k\cot kb \tag{7}$$

where

$$\begin{aligned}k_0&=\sqrt{\kappa_0^2-\zeta^2}\\ k&=\sqrt{\kappa_1^2-\zeta^2}.\end{aligned} \tag{8}$$

These functions will be used in the subsequent analyses. Moreover, the normalization factor is defined by

$$N(\zeta_s)=\left(\frac{2j\omega\mu_0k_0\sin^2 kb}{\zeta(1+jk_0b)}\right)^{1/2}_{\zeta=\zeta_s} \tag{9}$$

where unit incidence from the left is assumed.

The total (t) field are given by the sum of the incident (i) and scattered (s) fields as follows:

$$(E^t,H^t)=(E^i,H^i)+(E^s,H^s). \tag{10}$$

According to this definition, the boundary conditions can be summarized as follows:

(B1) radiation condition,
(B2) continuity of E_y^s and H_z^s at $x=b$,
(B3) continuity of E_y^t at the apertures of the notches (otherwise $E_y^t=0$ at $x=0$),
(B4) continuity of H_z^t at the aperture of the notches.

To summarize, the present problem is to find the scattered fields which obey (2)–(4) subject to the boundary conditions (B1)–(B4).

III. Spectral-Domain Analysis

In the following discussions, we use the Fourier and its inverse transformation of the form

$$\begin{aligned}f(\zeta)&=\int_{-\infty}^{\infty}f(z)e^{j\zeta z}\,dz\\ f(z)&=\frac{1}{2\pi}\int_c f(\zeta)e^{-j\zeta z}\,d\zeta\end{aligned} \tag{11}$$

where $f(z)$ is the original function and $f(\zeta)$ is its Fourier transform. The infinite contour c of the inverse Fourier transformation should be in the strip ($\operatorname{Im}\kappa_0<\zeta<-\operatorname{Im}\kappa_0$) resulting from the assumption that free space has a vanishingly small loss ($\operatorname{Im}\kappa_0<0$). However, we let $\operatorname{Im}\kappa_0\to 0$ when the analyses are completed.

Direct Fourier transformation of (2) leads to an ordinary differential wave equation for which solutions are well known. After straightforward but somewhat lengthy manipulations, we can express the Fourier transform of the scattered field above the ground plane in the form

$$\begin{aligned}E_y^s(x,\zeta)&=E_y^s(0,\zeta)\frac{k}{\sin kbG(\zeta)}e^{-jk_0(x-b)} \qquad (x>b)\\ &=E_y^s(0,\zeta)\left[\frac{k\sin kx}{\sin^2 kbG(\zeta)}+\frac{\sin k(b-x)}{\sin kb}\right] \qquad (0<x<b).\end{aligned} \tag{12}$$

It should be noted that (12) satisfies boundary conditions (B1) and (B2); (B1) may be satisfied if we choose the branch of k_0 appropriately, and (B2) can be conformed directly from (12) and its derivative with respect to the x coordinate in connection with (4).

Now, expanding the fields in the notches in terms of waveguide modes with unknown coefficients $A_{n\nu}$, we define the total electric field for $x<0$ by

$$\begin{aligned}E_y^t(x,z)&=e^{-j\zeta_1 nD}\sum_{\nu=1}^{\infty}j^{\nu-1}A_{n\nu}\sin[w_\nu(z-nD+w)]\\ &\quad\times\frac{\sin\gamma_\nu(d+x)}{\sin\gamma_\nu d}\\ &\qquad\text{for } -d<x<0,\ nD-w<z<nD+w,\\ &\qquad\text{and}\quad n=0,1,2,\cdots,N-1\\ &=0 \qquad \text{otherwise}\end{aligned} \tag{13}$$

where the term $e^{-j\zeta_1 nD}$ has been added only for computational convenience, and

$$\begin{aligned}w_\nu&=\nu\pi/2w \qquad (\nu=1,2,3,\cdots)\\ \gamma_\nu&=\sqrt{\kappa_2^2-w_\nu^2}.\end{aligned} \tag{14}$$

Then, the Fourier transform of (13) is given by

$$E_y^t(x,\zeta)=w\sum_{n=0}^{N-1}e^{j(\zeta-\zeta_1)nD}\sum_{\nu=1}^{\infty}A_{n\nu}F_\nu(\zeta)\frac{\sin\gamma_\nu(d+x)}{\sin\gamma_\nu d} \qquad (-d<x<0) \tag{15}$$

where

$$F_\nu(\zeta) = \nu\pi \frac{\sin(\zeta w - \nu\pi/2)}{\left[(\zeta w)^2 - (\nu\pi/2)^2\right]}. \tag{16}$$

It is worth noting that $E_y^s(0,\zeta)$ used in (12) should be equal to (15) when $x = 0$; that is,

$$E_y^s(0,\zeta) = w \sum_{n=0}^{N-1} e^{j(\zeta-\zeta_1)nD} \sum_{\nu=1}^{\infty} A_{n\nu} F_\nu(\zeta). \tag{17}$$

This relation ensures boundary condition (B3) because the electric field of the incident surface wave always vanishes on the ground plane.

We employ here the spectral-domain method in order to satisfy the remaining boundary condition (B4). From (4), the Fourier transform of the tangential component of magnetic fields for $x > 0$ and $x < 0$ is given by the derivative of respectively, (12) and (15) with respect to the x coordinate. On the other hand, that corresponing to only one notch aperture can be obtained by the convolution of the above-mentioned Fourier transform and a sampling function. As a result, (B4) regarding the mth notch aperture is expressed in the spectral domain as follows:

$$\frac{1}{\pi} e^{j\zeta mD} \int_c \frac{\partial}{\partial x}\left[E_y^s(+0,t) - E_y^t(-0,t)\right] S(t-\zeta) e^{-jtmD}\, dt$$
$$= -\int_{mD-w}^{mD+w} \frac{\partial}{\partial x} E_y^i(0,z) e^{j\zeta z}\, dz \qquad (m = 0,1,\cdots,N-1) \tag{18}$$

where the sampling function is defined by

$$S(x) = \frac{\sin xw}{x}. \tag{19}$$

Combining (5), (12), and (15) with (18), we can rearrange (18) in the form

$$\frac{1}{\pi} \sum_{n=0}^{N-1} e^{-j\zeta_1(n-m)D} \sum_{\nu=1}^{\infty} A_{n\nu} \int_c \tilde{G}(t) F_\nu(t) S(t-\zeta) e^{jt(n-m)D}\, dt$$
$$- \sum_{\nu=1}^{\infty} A_{m\nu} B_\nu F_\nu(\zeta) = -2 \frac{N(\zeta)k}{\sin kb}\bigg|_{\zeta=\zeta_1} S(\zeta-\zeta_1) \tag{20}$$

where

$$\tilde{G}(\zeta) = \frac{wk^2}{G(\zeta)\sin^2 kb} - kw \cot kb$$
$$B_\nu = w\gamma_\nu \cot \gamma_\nu d. \tag{21}$$

Thus, the present problem can be said to be solved if the unknown model coefficients are determined so that (20) may be satisfied.

IV. Inverse Fourier Transformation

According to the sampling theorem, (20) can be discretized in the following manner. First, substitute $\zeta = \pm w_\mu$ into (20). Second, add and subtract the resulting two equations when μ is odd and even, respectively. Then, after some algebraic manipulations, we have

$$\sum_{n=0}^{N-1} e^{-j\zeta_1(n-m)D} \sum_{\nu=1}^{\infty} A_{n\nu} I(m,n;\mu,\nu) - 2\sum_{\nu=1}^{\infty} A_{n\nu} B_\nu \delta_{\mu\nu}$$
$$= -2N(\zeta)\frac{kw}{\sin kb} F_\mu(\zeta)|_{\zeta=\zeta_1} \qquad (m = 0,1,\cdots,N-1) \tag{22}$$

where $\delta_{\mu\nu}$ is the Kronecker δ, and

$$I(m,n;\mu,\nu) = -2jw \sum_{s=1}^{M} \mathrm{Res}\left[\tilde{G}(\zeta_s)\right] F_\mu(\xi_s) F_\nu(\zeta_s) \left\{ {\cos \atop j\sin} \right\}$$
$$\times \zeta_s(n-m)D + 2\frac{w}{\pi} p\cdot v\cdot \int_0^\infty \tilde{G}(t) F_\mu(t)$$
$$\times F_\nu(t) \left\{ {\cos \atop j\sin} \right\} t(n-m)D\, dt \tag{23}$$

where cos and $j\sin$ correspond to the cases where $\nu + \mu$ is even and odd, respectively. In this derivation, we have finally let $\mathrm{Im}\,\kappa_0 \to 0$, and so residue calculus and Cauchy's principal value have appeared. In general, integration in (23) should be performed numerically. For $|n-m|\kappa_0 D \gg 1$, however, it can be calculated with good accuracy by means of the saddle point method (see Appendix I).

Since (22) constitutes infinite sets of algebraic equations with respect to the unknown modal expansion coefficients $A_{n\nu}$, we can solve these linear equations numerically by truncating the modal numbers appropriately. In numerical calculations, however, a successive iterative method is needed in order to reduce the dimension of matrices, especially for a large number of notches. Once these coefficients are determined, all the physical quantities can be obtained by applying the Fourier inverse transformation to (12) together with (17).

By use of residue calculus, the reflection and transmission coefficients of the excited surface waves on the left and right waveguide, respectively, are given by

$$R_s = -j\frac{E_y^s(0,-\zeta_s)}{N(\zeta_s)\zeta_s} \cdot \frac{k^2 \cos kb \sin kb}{[kb\cos kb - \sin kb]}\bigg|_{\zeta=\zeta_s} \tag{24}$$

$$T_s = -j\frac{E_y^s(0,\zeta_s)}{N(\zeta_s)\zeta_s} \cdot \frac{k^2 \cos kb \sin kb}{[kb\cos kb - \sin kb]}\bigg|_{\zeta=\zeta_s} \qquad (s = 1,2,\cdots,M). \tag{25}$$

On the other hand, the radiative far field is easily calculated by applying the saddle point method to the inverse Fourier transformation. As a result, the radiation power density is defined by

$$P(\theta) = \sin^2\theta \frac{\kappa_0^2}{2\pi} \left| \frac{E_y^s(0,\zeta)k}{G(\zeta)\sin kb} \right|^2_{\zeta-\kappa_0\cos\theta} \tag{26}$$

where θ indicates the angle of the observation point with respect to the z axis. The total radiation power is related to the aforementioned density by

$$R_{\mathrm{rad}} = \int_0^\pi P(\theta)\, d\theta. \tag{27}$$

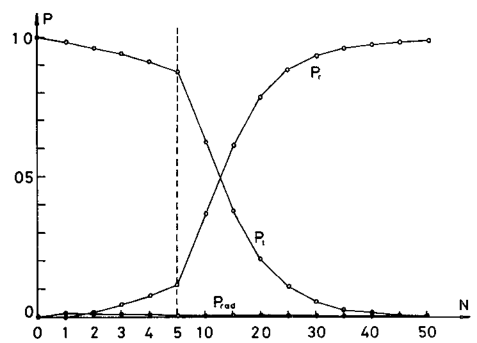

Fig. 2. Reflected, transmitted, and radiated powers versus the number N of the notches near Bragg reflection. $\kappa_0 b = 1.97$, $\epsilon_1/\epsilon_0 = \epsilon_2/\epsilon_0 = 3.0$, $D/b = 1.1859$, $d/w = 0.8008$, and $w/D = 0.25$.

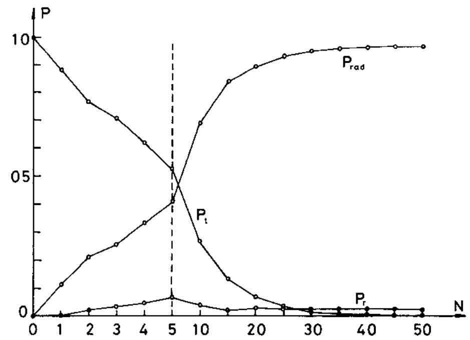

Fig. 3. Radiated, transmitted, and reflected powers versus the number N of the notches near maximum radiation. $\kappa_0 b = 3.26$, $\epsilon_1/\epsilon_0 = \epsilon_2/\epsilon_0 = 3.0$, $D/b = 1.1859$, $d/w = 0.8008$, and $w/D = 0.25$.

Since the unit incidence from the left has been considered, the power relation should be

$$\sum_{s=1}^{M} |R_s|^2 + \sum_{s=1}^{M} |T_s|^2 + P_{\text{rad}} = 1 \tag{28}$$

where the first and the second term denote the total reflection and the transmission power, respectively.

V. Numerical Examples

As mentioned in the Introduction, the investigation of surface-wave scattering by a finite periodic structure is important in connection with mode filters as well as leaky wave antennas. In this section, we show some numerical results. Fig. 2 shows the variation of the reflected power P_r, transmitted power P_t, and radiated power P_{rad} versus the number N of notches. The parameters with respect to the dimension are chosen such that when the normalized frequency is selected as $\kappa_0 b = 2.0$, the Bragg reflection condition based on first-order perturbation theory [1] may be satisfied, that is, $\zeta_1 D = \pi$, and each notch may behave as a quarter-wave matching circuit, that is, $\gamma_1 d = \pi/4$. Although the normalized frequency is somewhat different from the above-mentioned point ($\kappa_0 b = 1.97 \neq 2.0$), maximum reflection is observed. This is why the phase constant of the mode supported on the periodic structure [12] satisfies the condition, that is, $\beta_0 D = \pi$, as described later. The reflected power increases monotonically with the number of notches, amounting to 98.87 percent for $N = 50$, whereas the transmitted and radiated powers are very small, 0.36 percent and 0.75 percent, respectively.

Fig. 3 shows the variation of the radiated, transmitted, and reflected powers versus the number of notches. The parameters used are much the same as those in Fig. 2 except for the normalized frequency; that is, $\kappa_0 b = 3.26$. At this normalized frequency, as described later, the periodically notched dielectric guide can support a surface wave mode with a phase constant for which the Bragg radiation condition, $\beta_0 D = \kappa_0 D \cos\theta + 2\pi$, is satisfied for real θ. When $N = 50$, the radiation power amount to 96.94 percent, while the transmitted and reflected powers are, respectively, 0.13 percent and 2.93 percent.

Fig. 4 shows the variation of the moduli of the reflection and transmission coefficients versus the normalized frequency. It is seen that the greater the number of notches, the more sensitive the reflectivity becomes with respect to the normalized frequency. The parameters used are chosen to satisfy the Bragg condition based on first-order perturbation theory at $\kappa_0 b = 2.0$, as shown in Fig. 2. However, numerical results show that maximum reflection oc-

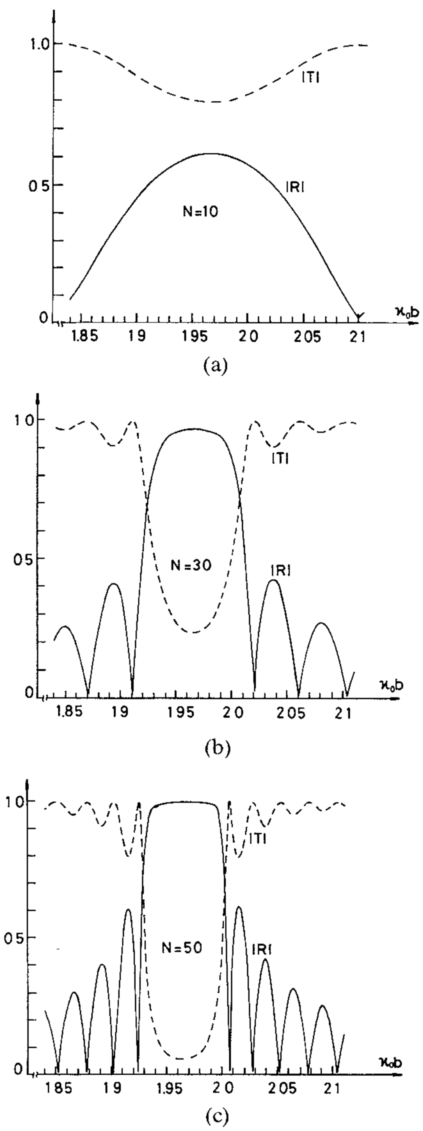

Fig. 4. Reflection and transmission coefficients versus the normalized frequency $\kappa_0 b$ near Bragg reflection for (a) $N = 10$, (b) $N = 30$, and (c) $N = 50$. $\epsilon_1/\epsilon_0 = \epsilon_2/\epsilon_0 = 3.0$, $D/b = 1.1859$, $d/w = 0.8008$, and $w/D = 0.25$.

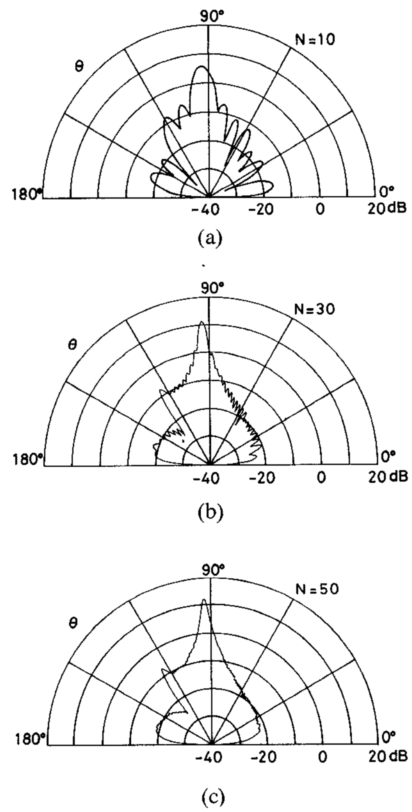

Fig. 5. Radiation power density versus observation angle θ near maximum radiation for (a) $N=10$, (b) $N=30$, and (c) $N=50$. $\kappa_0 b=3.26$, $\epsilon_1/\epsilon_0=\epsilon_2/\epsilon_0=3.0$, $D/b=1.1859$, $d/w=0.8008$, and $w/D=0.25$.

curs at a slightly different frequency. This noticeable feature may be considered to reflect the fact that the corrugation is so strong that the Bragg condition based on first-order perturbation theory does not hold for these numerical examples. For example, when $\kappa_0 b=1.97$, the propagation constant of the perturbed waveguide mode is calculated as $\zeta_0 D=3.1415-0.0586j$ (see Appendix II), whereas that of the unperturbed one is $\zeta_1 D=3.0732$. Thus, we can closely estimate the maximum reflection point by applying the Bragg condition to the phase constant β_0 rather than ζ_1.

Fig. 5 show the variation of the radiation power density $P(\theta)$ versus the observation angle θ. The parameters used are coincident with those in Fig. 3. In this case, the propagation constants of the unperturbed and perturbed waveguides are given by $\zeta_1 D=5.9719$ and $\zeta_0 D=6.0323-0.0128j$, respectively. As a result, the Bragg condition reveals that using ζ_1 in an approximate sense, maximum radiation occurs at $\theta \cong 94.6°$, whereas using β_0, it occurs at $\theta \cong 93.7°$. In fact, numerical results show that it takes place at $\theta \cong 93.0°$ with the peak power density $P(\theta)\cong 16.34$ ($\cong 12.12$ dB) for $N=50$. It is found from the numerical examples that we can estimate the direction of the maximum radiation more accurately by using not the phase constant ζ_1 but β_0.

VI. Conclusions

We have analyzed rigorously the surface-wave scattering by finite periodic notches loaded in a ground plane. The analytical method is based on the spectral-domain analysis combined with the sampling theorem. In the numerical calculations, we have employed an iterative method in order to reduce the dimension of the matrices, enabling us to treat a fairly large number of notches. Numerical calculations were made concentrating on the maximum reflection or radiation. In all the numerical examples given, the power relation (28), which can be considered a check of the accuracy, was fulfilled to within 0.5 percent. From the numerical results, it has been found, that the Bragg condition based on first-order perturbation theory cannot predict the maximum reflection or radiation precisely because of the strong corrugation.

In this paper, we have treated only TE excitation, but the method can also be applied to the TM case. This problem deserves further attention.

Appendix I
Saddle Point Method

When $(m-n)\kappa_0 D \gg 1$, application of the saddle point method to (23) yields

$$I(m,n;\mu,\nu) \cong -2jw\sum_{s=1}^{M}\operatorname{Res}\left[\tilde{G}(\zeta_s)\right]F_\mu(\zeta_s)F_\nu(\zeta_s) \times e^{-j\zeta_s(m-n)D} + 2j\sqrt{2\kappa_0 w}\left(\kappa_1^2-\kappa_0^2\right)w^2 \times F_\mu(\kappa_0)F_\nu(\kappa_0)e^{-2j\kappa_0 w} \times \tilde{I}(\xi,(m-n)D/w-2) \tag{A1}$$

where

$$\xi = w\sqrt{\kappa_0^2\sin^2\kappa_1 b+\kappa_1^2\cos^2\kappa_1 b}. \tag{A2}$$

The integral $\tilde{I}$ along the branch cut is given by

$$\tilde{I}(A,B)=\frac{(j-1)}{2A}\left\{\sqrt{\kappa_0 w-A}\,\tilde{F}\left(\sqrt{2B(\kappa_0 w-A)/\pi}\right)e^{-jAB} - \sqrt{\kappa_0 w+A}\,\tilde{F}\left(\sqrt{2B(\kappa_0 w+A)/\pi}\right)e^{jAB}\right\} \tag{A3}$$

where we have used the Fresnel function of the form

$$\tilde{F}(x)=\int_x^\infty e^{-j\pi t^2/2}\,dt. \tag{A4}$$

It should be pointed out that (A1) gives us an accurate value even near a cutoff frequency.

When $(n-m)\kappa_0 D \gg 1$, the final result can be expressed by use of (A1) as follows:

$$I(m,n;\mu,\nu)=(-1)^{\mu+\nu}I(n,m;\mu,\nu). \tag{A5}$$

Appendix II
Characteristic Equation

The propagation constant ζ_0 of the mode supported on a periodically notched waveguide is given by the zero of the characteristic equation which is formally obtained from (20) in the following way. Let the right-hand side of (20) be zero, put $\zeta_1 \to \zeta_0$, and consider the summation with respect to n from $-\infty$ to $+\infty$. Then, by use of the

Poisson summation formula, we have

$$\frac{2}{D}\sum_{\nu=1}^{\infty} A_\nu \sum_{n=-\infty}^{\infty} \tilde{G}(\zeta_0+D_n)F_\nu(\zeta_0+D_n)S(\zeta_0+D_n-\zeta) - \sum_{\nu=1}^{\infty} A_\nu B_\nu F_\nu(\zeta)=0 \quad \text{(A6)}$$

where Floquet's theorem has been taken into account; that is, $A_{n\nu} \to A_\nu$ independent of the notch number n, and

$$D_n = 2\pi n/D. \quad \text{(A7)}$$

Based on the sampling theorem, (A6) can be discretized in the same way as the derivation of (22) as follows:

$$\sum_{\nu=1}^{\infty}\left[\frac{w}{D}\sum_{n=-\infty}^{\infty}\tilde{G}(\zeta_0+D_n)F_\mu(\zeta_0+D_n) \times F_\nu(\zeta_0+D_n) - B_\nu\delta_{\mu\nu}\right]A_\nu = 0 \qquad (\mu=1,2,3,\cdots). \quad \text{(A8)}$$

As a result, ζ_0 can be determined in such a way that the determinant of (A8) should be zero. In general, ζ_0 is given by a complex number; its real part is the phase constant β_0 of the mode, and the imaginary part corresponds to the attenuation constant.

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References

[1] D. Marcuse, *Light Transmission Optics*. New York: Van Nostrand Reinhold, New York, 1982.

[2] H. Shigesawa, M. Tsuji, and K. Takiyama, "Dielectric gratings as circuit components in MM and SUBMM-wave region," in *9th Int. Conf. IR MM Waves* (Takaruzaka, Japan), Oct. 1984, paper T-9-7.

[3] K. Uchida, "Surface wave scattering by finite periodic notches," in *1985 Int. Symp. Antennas Propagat.* (Kyoto, Japan), Aug. 1985, paper 142-2.

[4] H. Shigesawa and M. Tsuji, "Microwave network approach to dielectric periodic leaky-wave antennas," in *1985 Int. Symp. Antennas Propagat.*, (Kyoto, Japan), Aug. 1985, paper 022-5.

[5] H. Shigesawa and M. Tsuji, "A completely theoretical design method of dielectric image guide gratings in the Bragg reflection region," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 420–426, Apr. 1986.

[6] R. D. Nevels and C. M. Butler, "Electromagnetic scattering by a surface wave from a notch in a ground plane covered by a dielectric slab," *J. Appl. Phys.*, vol. 52, no. 5, pp. 3145–3147, May 1981.

[7] K. Solbach, "Slots in dielectric image line as mode launchers and circuit elements," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 10–16, Jan. 1981.

[8] W. S. Park and S. R. Seshadri, "Reradiation from a grating coupler for a grounded dielectric slab waveguide," *Proc. Inst. Elec. Eng.*, vol. 133, no. 1, pp. 10–17, Feb. 1986.

[9] K. Uchida, "A consideration on application of Fourier transformation to electromagnetic field boundary-value problem involving infinite length," *Trans. IECE Japan*, vol. J68-B, pp. 1215–1216, Oct. 1985.

[10] K. Uchida, T. Noda, and T. Matsunaga, "Numerical analysis of plane wave scattering by an infinite plane grating using weighted Fourier series," *Trans. IECE Japan*, vol. E69, pp. 132–138, Feb. 1986.

[11] K. Uchida, T. Noda, and T. Matsunaga, "Spectral domain analysis of electromagnetic wave scattering by an infinite plane metallic grating," to be published.

[12] S. T. Peng, T. Tamir, and H. L. Bertoni, "Theory of periodic waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 123–133, Jan. 1975.

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